**Self-assessment answers: 14 Lines and planes in space**

**1.** (a) Direction vector is , so the vector equation of l1 is  (or equivalent).

In Cartesian form, this is *x* – 4 = −*y* = .

(b) Intersection where 

⇒ 

(1) + (2) ⇒ 4 = 1 + *t*

⇒ *t* = 3

(2) ⇒ *λ* = −2

(3) ⇒ 3 + 4 = −5 + 6: False

These are skew lines and do not intersect. *[8 marks]*

**2.** (a) Normal vectors are  and .

Angle between planes is the same as angle between normals.

***n*1** **·** ***n*2** **=** |***n*1**||***n*2**| cos*θ*

⇒ *θ* = arccos

= arccos

= 1.09 radians (62.4°) (3SF)

(b) Direction vector ***d*** of intersecting line is perpendicular to both normals:

***d* = *n*2 × *n*1** = 

A point on both lines (by inspection) is (2, 0, 1).

So the vector equation of the intersecting line is  *[8 marks]*

**3.** (a) If ,

⇒ 

(1) ⇒ = −2

(2) ⇒ 1 + *t* = −1 which is consistent

(3) ⇒ 1 + 2*t* = −3 which is consistent

So (5, −1, −3) is on l.

(b) 

⇒ 

(c) Normal to the plane , plane passes through (5, −1, −3).

So the Cartesian equation of the plane is *x* – 7*y* + 4*z* = 0.

(d) P lies on l, so P =  for some *t*. Then .

Require DP perpendicular to , so 

⇒ (*t* + 2) + (*t* – 2) + (4*t* – 6) = 0

⇒ *t* = 1

⇒ P = (2, 2, 3) *[14 marks]*